AIAA JOURNAL
Vol. 35, No. 5, May 1997

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Novel Method for Calculating Two-Dimensional Blade Vortex Interaction

Ronald J. Epstein,* John A. Rule,† and Donald B. Bliss‡
Duke University, Durham, North Carolina 27708-0300

Introduction

THE interaction of a helicopter rotor with one or more of its own shed tip vortices is a common aeroacoustic phenomenon of considerable interest. These interactions, known as blade–vortex interactions (BVI), can be responsible for much of the vibratory airloading on a rotorblade. Many different BVI geometries are possible; however, parallel BVI, which occurs when the rotorblade and impinging vortex are very close and nearly parallel, is of particular interest because it has been experimentally shown to be a major source of rotorcraft noise.[1]

A large number of aerodynamic and acoustic calculations have been made, which embody a wide range of BVI models. The BVI model typically consists of an aerodynamic analysis coupled with an acoustic prediction scheme, as a postprocess.[2] The aerodynamic analyses can vary in complexity from an idealized, two-dimensional, incompressible, inviscid flow model, to fully three-dimensional, viscous, compressible, computational fluid dynamics (CFD) techniques.[3] The acoustic prediction methods used in BVI calculations are usually of two types: acoustic analogy methods[4–6] and Kirchhoff methods.[7–9] Therefore, to calculate the acoustic field radiated by a BVI, the aerodynamic model must be coupled with the acoustic model in a consistent fashion. The combined aeroacoustic methodology can become numerically cumbersome because of the calculations needed to solve both the aerodynamic and acoustic problems. Taken together these calculations can require an inordinate amount of computer time, especially when considering the higher acoustic frequencies, which are required to fully resolve the radiated acoustic field of a BVI.

The present research involves an innovative method for calculating two-dimensional BVI. The methodology involves the application of a newly developed aeroacoustic boundary element method, which uses analytical/numerical matching (ANM), in conjunction with a novel vortex method, turbulent core model (TCM) vortex dynamics. ANM is a hybrid scheme combining a low-resolution global numerical solution with high-resolution local solutions to form a composite solution. ANM provides high-resolution calculations from low-resolution numerics with analytical corrections, while avoiding the subtlety involving singular integral equations and their numerical implementation. TCM vortex dynamics is based on the fundamental integral conservation laws of the aerodynamic flowfield and offers a fully self-consistent real flow vortex model, which has no empirical free parameters. Together, these models offer a fresh point of view on BVI modeling, in the framework of a fully consistent and unified aeroacoustic theory.

For the purposes of illustration, this Note presents results for an idealized BVI calculation. The far-field acoustic radiation from a flat plate airfoil encountering a periodic row of TCM vortices is presented. It is expected that with further development of the methodology, future calculations will include more complex and realistic configurations.

Vortex Model

The present calculation makes use of a new vortex model, which calculates viscous vortex structure from prescribed loading parameters on a vortex generating aerodynamic body. This is a general method, which applies to both independently generated vortices, such as would result from a fixed wing or a wind-tunnel vortex generator, and self-generated vortices, which would be trailed from a lifting rotor blade. The model does not make use of curve fits or empirically determined laws but instead relies on a set of fundamental conservation laws and basic aerodynamic principles.

The theoretical basis for the present vortex model lies with the Betz[10] method for vortex sheet rollup. Betz established the following unique set of global integral invariants for an unbounded, inviscid, two-dimensional vorticity distribution: circulation, centroid of vorticity, and second moment of vorticity about the centroid. These integrated quantities are invariant with time and can be expressed in differential form and applied to an initially flat sheet of vorticity. This sheet will roll up on a station-by-station basis into an axisymmetric trailing vortex.

Until recently, rollup models based on the Betz[10] method suffered from the shortcoming of singular velocities in the center, or core region, of the predicted vortex. These singularities result from two factors: singular vorticity at the outboard edge of the initial vortex sheet for most cases of interest and the assumption that the rollup process is completely inviscid. However, experimental vortex studies suggest that viscous effects are important only locally in the vortex core region and the Betz assumption of inviscid flow is reasonable outside the core.[11]

The present model establishes the length scale in the core region, generally termed the core radius, over which viscous effects are important. It also provides a set of conservation laws, which produce bounded, experimentally consistent velocity distributions free of Betz type singularities, using a well-established vortex turbulent mixing theory.[12] The details of this new vortex model may be found in Ref. 12, and the essentials are outlined subsequently.

Previous work has demonstrated that three-dimensional, inviscid vortex sheet rollup conserves three quantities at every intermediate stage of rollup: mass flux, axial flux of angular momentum, and axial momentum flux through the use of Bernoulli's equation.[13] The first two quantities are directly related to the original integral invariants established by Betz. In the core region, viscous effects will alter the vortex spiral structure by means of turbulent mixing. Theoretical and experimental vortex studies have conclusively shown that turbulent mixing will produce a laminarescent, or well-organized, axisymmetric vortex core of solid body rotation, surrounded by a transition region where circulation is proportional to the logarithm

Received Dec. 15, 1995; revision received Aug. 12, 1996; accepted for publication Dec. 2, 1996.

*Research Assistant Professor, Department of Mechanical Engineering and Material Science; currently Project Engineer, Advanced Systems and Technology, McDonnell Douglas Aerospace, St. Louis, MO 63166-0516. Member AIAA.

†Research Assistant, NASA Graduate Student Researchers Program Fellow, Department of Mechanical Engineering and Material Science. Member AIAA.

‡Associate Professor, Department of Mechanical Engineering and Material Science. Member AIAA.

of radius. This two-region core is surrounded by an inviscid spiral structure consistent with that predicted by modified Betz theory.

Within the turbulent core region, the set of conservation relations just listed is applied on an integral basis rather than a differential basis. Quantities are conserved between the initial flat sheet of vorticity and a downstream trailing vortex, which has core velocity profiles with prescribed functional forms consistent with turbulent mixing. The solution profiles have free constants, which are uniquely determined as a function of prescribed upstream conditions. Viscous effects are handled implicitly by the particular functional forms chosen, and so viscosity and Reynolds stress do not appear anywhere in the formulation. This procedure bears a similarity to the von Kármán–Pohlhausen method for boundary layers.

This new approach to vortex rollup modeling calculates steady-state vortex properties directly from a prescribed circulation distribution. Vortex solutions contain no free parameters, eliminating the need to arbitrarily assign a core size or circulation to the vortex. The method is robust and has been shown to compare well with experiment.[13]

Aeroacoustic Model

In the present analysis, the periodic nature of BVI is modeled by a flat plate moving at constant velocity through a fixed infinite row of TCM vortices. An exact solution exists for the upwash due to an infinite row of potential vortices.[14] Because TCM vortices return to the potential result outside the inviscid spiral region, it is possible to calculate the upwash due to an infinite row of TCM vortices by locally removing the upwash due to a single potential vortex and adding in the upwash due to a single TCM vortex. This local correction is only made for the vortex in the immediate vicinity of the flat plate.

Once the upwash due to an infinite row of TCM vortices is calculated, this upwash field is decomposed into a Fourier sine series, so that aeroacoustic calculations are necessary only for purely harmonic upwash inputs. The frequencies in the sine series are a function of vortex spacing alone, and the problem is linear. Thus, the acoustic radiation due to a series of modes with unit input amplitude can be calculated once. Modal coefficients can be calculated for any input vortex structure or strength, and the resulting acoustic pressures due to a particular case are calculated as a postprocess by summing up the necessary modes with the proper weighting coefficients.

In general, the governing equation of linear, compressible, unsteady potential flow can be written[15,16]

$$\left(1-\frac{U_\infty^2}{a_\infty^2}\right)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}-2\frac{U_\infty}{a_\infty^2}\frac{\partial^2\varphi}{\partial x\partial t}-\frac{1}{a_\infty^2}\frac{\partial^2\varphi}{\partial t^2}=0 \qquad (1)$$

where U_∞ is the freestream velocity, a_∞ is the speed of sound, t is time, and φ is the velocity potential. Recognizing that the solution to Eq. (1) can be written in terms of a jump in potential $\Delta\varphi$ on the boundary S (Refs. 15, 17–21), assuming a harmonic time dependence, and denoting the representative source and doublet strengths, respectively, as σ and μ, the solution can be written as

$$\bar{\varphi}(x,y,z)=\frac{1}{4\pi}\int_S\left\{\sigma\frac{e^{-i\omega\tau}}{R}-\mu\frac{\partial}{\partial n}\frac{e^{-i\omega\tau}}{R}\right\}\mathrm{d}S \qquad (2)$$

where $\varphi=\bar{\varphi}e^{i\omega t}$ and $\tau=D/a_\infty$, and the time factor is suppressed.

Evaluating Eq. (2) on the surface S produces an integral equation for $\bar{\varphi}$. Therefore, when considering an arbitrary lifting surface in an infinite medium, we take S to be the surface of the body and the wake. The boundary conditions can be specified as a Neumann problem, a Dirichlet problem, or some combination of the two. Here, a Neumann boundary condition is assumed. This simply requires taking the normal derivative of Eq. (2), thereby recasting the problem in terms of the known normal wash specified on the surface S and the unknowns σ and μ.

The ANM smoothed global solution is constructed by an overlapping distribution of smoothed doublet solutions to Eq. (1). A smoothed doublet is analogous to the familiar singular doublet solution for compressible unsteady flow, except there is a smooth spatial distribution of doublet strength density.[22–24] Smoothed doublets are nonsingular everywhere, and away from the smoothing region, they have the same behavior as an equivalent singular doublet with the same aggregate strength. The smooth doublet is denoted by $R \to R_s$, where this transformation can be simply applied to Eq. (2). Overlapping discrete smoothed doublets have been shown to approximate a continuous distribution accurately.[22–24] Distributing overlapping smoothed doublets over an aerodynamic surface and its associated wake is an efficient way to calculate a low-resolution smoothed global solution. However, this global solution must be corrected by the addition of local solutions to recover the correct answer to the original problem.

The ANM local solution[22–26] is the difference between a high-resolution local solution based on singular doublets and a corresponding smoothed local solution, called the matching solution, based on smoothed doublets. The difference between these solutions cancels far away, because they have the same far-field behavior. Therefore, the far-field shape and behavior is unimportant, allowing very simple solution forms to be used. Appropriate high-resolution local solutions on surfaces and near edges can be found by separation of variables, perturbation methods, or by direct integration over singular doublet distributions of infinite or semi-infinite extent. Matching solutions can found by integration over corresponding smoothed doublet distributions, or by a special variable separation technique combined with an equivalent singularity displacement method called the split sheet analogy.[22,24]

The composite solution is formed from the low-resolution global solution, plus the high-resolution local solution $\mathcal{L}$ in Eq. (3), minus the low-resolution local matching solution $\mathcal{M}$. As Eq. (3) indicates, the local solutions scale with doublet strength, as does the global solution,

$$\frac{\partial\bar{\varphi}}{\partial n}=-\frac{1}{4\pi}\int_S\left\{\mu\frac{\partial^2}{\partial n^2}\frac{e^{-i\omega\tau}}{R_s}\right\}\mathrm{d}S+\mu\mathcal{L}-\mu\mathcal{M} \qquad (3)$$

At a point of evaluation on the surface, the global solution provides the correct far-field contribution, but its near-field contribution is smoothed. Furthermore, the high-resolution local solution and the matching solution cancel in the far field, leaving the global contribution. Therefore, taken together, the local solutions are nonradiating. When done properly, the final answer is independent of the degree of smoothing over a wide range of smoothing length scale values. This independence demonstrates that a region of solution overlap has been achieved. Further details about the derivation, and functional form of the local and matching solutions are given in Refs. 22–26.

Numerical Results and Discussion

Figure 1 shows a schematic diagram of the model BVI calculation for a flat plate passing through an infinite row of two-dimensional TCM vortices. The plate passes through the center of the vortices. The vortex centers are separated in space by 5 chord lengths, and the flat plate is traveling at $M = 0.5$, where $M = U_{\text{plate}}/a_\infty$, relative to the acoustic observer in a fixed frame of reference. The observation point is chosen to be 20 chord lengths above the row of vortices. In time, the plate approaches the observation point, passes directly beneath it, and then travels away.

The relevant length scales in the problem are the spacing between vortices L, the chord of the plate $C = 2b$, the vertical miss distance H between the plate and the vortex center, the vortex core radius r_c, the circulation of the vortices Γ, the plate velocity U_∞, the speed of sound a_∞, the fluid density ρ, and the time t. These nine parameters can be appropriately reduced to six nondimensional groups. For the present calculation, the nondimensionalized acoustic perturbation pressure was calculated at the observer as a function of nondimensionalized time τ. The pressure is normalized by $\rho a_\infty(\Gamma/2\pi C)$, which represents the ratio of the pressure to the characteristic acoustic impedance multiplied by a characteristic velocity. The time is normalized by the semichord b and the plate velocity U_∞. This is the ratio of time to the characteristic convective time for an acoustic wave.

Figure 2a shows the results of the fully converged calculation using 75 acoustics modes. At $\tau = 0$, the plate is in front of the observation point. The plate passes through three vortices while approaching the observation point, and then at $\tau = 25$ the plate is below the observation point. The calculation shows the characteristic BVI signature with the impulsive pressure variation as a function

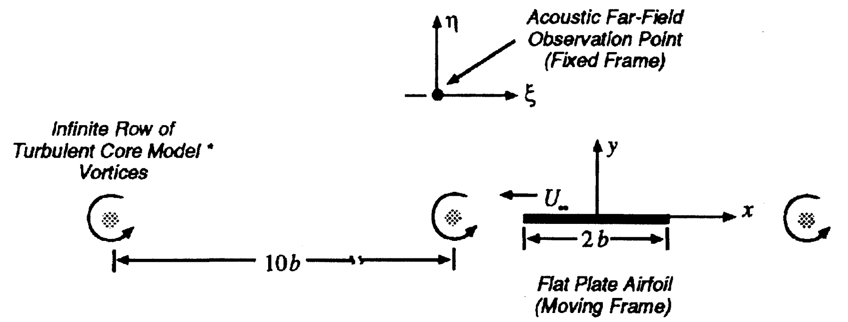
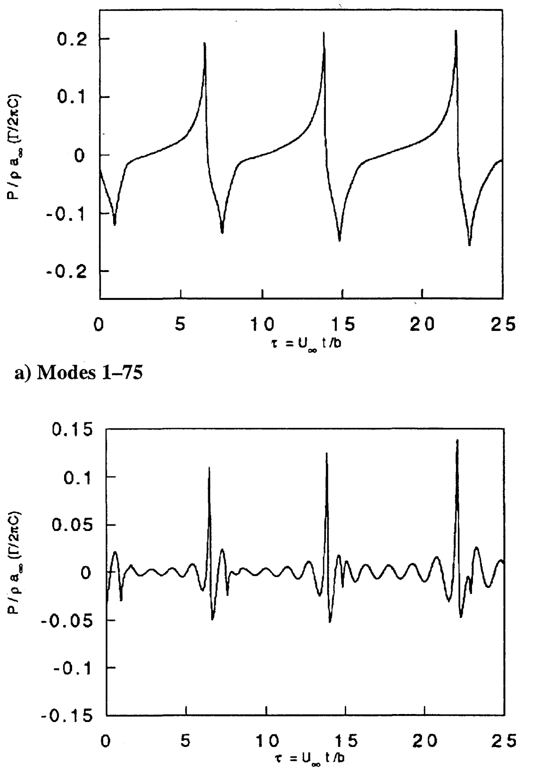


Fig. 1 Schematic representation of the two-dimensional flat plate BVI calculation, modeled with the ANM boundary element method and TCM vortices.



a) Modes 1–75

b) Modes 6–75

Fig. 2 Acoustic pressure calculation for a two-dimensional flat plate BVI with a row of TCM vortices; $L/C = 5$, $H/C = 0$, $r_c/C = 0.025$, and $M = 0.50$.

of time. Each sharp impulse represents the interaction of the plate and an impinging vortex. The peak of each pressure impulse increases as the relative location between the plate/vortex interaction and the observation point becomes smaller. Note that this is a two-dimensional calculation done with both a two-dimensional aerodynamic and acoustic model. Despite the idealized nature of the calculation, some physical trends of the aeroacoustic phenomena can be recognized. The effects of the circulation and plate chord can be easily seen from the results. If the circulation of the impinging vortices is increased, the radiated acoustic pressure will increase as well. This is expected because the impinging vortices act as the unsteady forcing to the aeroacoustic system. Similarly, if the characteristic acoustic impedance is increased, the radiated acoustic response will also increase. If the chord of the plate increases, the radiated acoustics will decrease, as the net acoustic source strength, which remains constant, is spread over a larger surface allowing for greater wave interaction and cancellation. It is interesting to consider the radiation from only modes 6–75, which effectively is a high-pass-filtered signal, shown in Fig. 2b. The figure clearly shows the impulsive nature of the interaction and the short time over which it occurs relative to the characteristic convective time scale in the flow.

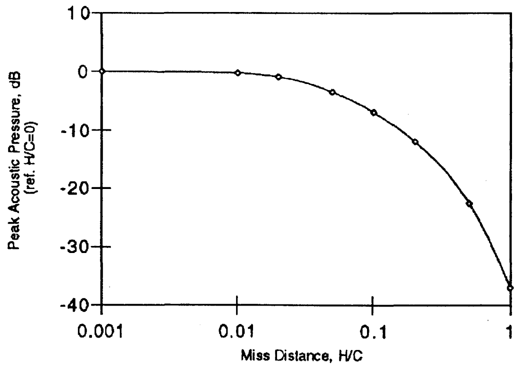


Fig. 3 Peak radiated acoustic pressure at observation point as a function of miss distance for a two-dimensional flat plate BVI with a row of TCM vortices; $L/C = 5$, $r_c/C = 0.025$, $M = 0.5$, and modes 1–75.

Figure 3 shows a parametric study of the effect of the nondimensionalized miss distance parameter H/C on the peak acoustic pressure measured at the observation point. The peak acoustic pressure is written in terms of decibels relative to the $H/C = 0$ interaction and plotted as a function of miss distance parameter. As the miss distance increases, the peak radiated acoustic pressure falls off rapidly. For example, Fig. 3 shows that a change in miss distance of 10% of the chord, which is roughly the thickness of a real wing section, corresponds to 6.8-dB fall in peak acoustic pressure in the field. This is approximately a fourfold decrease in the predicted peak pressure.

The implication of this result is far reaching. In fact, it means that for any BVI computer simulation, CFD or otherwise, the miss distance must be calculated accurately or the radiated acoustics will be rendered very inaccurately. In many problems of practical interest, the viscous core radius of the impinging vortex is on the order of the airfoil thickness. This calculation demonstrates that errors in the predicted core location or size will have a significant impact on the validity of acoustic predictions. Therefore, as Fig. 3 shows, the radiated acoustics from a BVI calculation are very sensitive to the miss distance H, and it follows that the smallest length scales in the problem are important, even those on the order of the airfoil thickness. This stresses the need for very accurate fluid modeling in order to achieve the desired accuracy.

Conclusion

A boundary element method based on smoothed fundamental solutions to the convective wave equation has been applied successfully to an idealized problem that models a two-dimensional BVI. A flat plate airfoil is passed through an infinite row of vortices with viscous cores, and the radiated acoustic pressure is calculated at a fixed location in the far field. This formulation is similar to the periodic encounter of a rotor blade with an independently generated vortex.

The aeroacoustic method used to solve this problem is based on ANM, which offers a fresh point of view on classical lifting surface theory by solving the problem by a fundamentally different means. The ANM approach not only offers a unified aeroacoustic method for lifting surface aerodynamics and acoustics but is also free from many of the problems traditionally associated with the numerical solution of singular integral equations. The ANM boundary element formulation leads to accurate solutions and rapid numerical convergence and is free from the ambiguity present in many traditional singularity methods.

The input gust field in this problem is composed of an infinite row of vortices with viscous central cores. Each component vortex in the row is based on a new analytical model, which calculates trailing vortex structure based on prescribed blade loading distribution and strength. In this model problem, vortex properties are calculated for an assumed elliptic lift distribution with net lift coefficient of unity. There are no free parameters, such as core size, in this vortex model. All vortex properties are determined uniquely as a result of the assumed generating lift distribution. Thus, the present vortex model is applicable to both independently generated and self-generated vortices in the study of BVI.

Calculations for the radiated acoustic pressure are given, which exhibit the behavior expected from a two-dimensional BVI, and the effect of varying the vortex miss distance on the peak radiated acoustic pressure is investigated.

Acknowledgments

The first author was supported by a Graduate Student Researchers Program Fellowship from the NASA Langley Research Center with Thomas F. Brooks serving as Technical Monitor. The second author is supported by a Graduate Student Researchers Program Fellowship from the NASA Ames Research Center with William G. Warmbrodt serving as Technical Monitor. The authors would like to thank Todd R. Quackenbush at Continuum Dynamics, Inc., for helpful discussions while the manuscript was in preparation.

References

[1]Caradonna, F. X., Laub, G. H., and Tung, C., "An Experimental Investigation of the Parallel Blade-Vortex Interaction," *Proceedings of the 10th European Rotorcraft Forum* (The Hague, The Netherlands), American Helicopter Society, Alexandria, VA, 1984.

[2]Prichard, D., Burley, C., and Boyd, D. D., Jr., "Computational Techniques for Improved Blade Vortex Interaction Modeling," *Proceedings of the 51st Annual Forum of the American Helicopter Society* (Fort Worth, TX), American Helicopter Society, Alexandria, VA, 1995.

[3]Kitaplioglu, C., and Caradonna, F. X., "Aerodynamics and Acoustics of Blade-Vortex Interaction Using an Independently Generated Vortex," *Proceedings of the American Helicopter Society Aeromechanics Specialists Conference* (San Francisco, CA), American Helicopter Society, Alexandria, VA, 1994.

[4]Farassat, F., and Brentner, K. S., "The Uses and Abuses of the Acoustic Analogy in Helicopter Rotor Noise Prediction," *Journal of the American Helicopter Society*, Vol. 33, Jan. 1988, pp. 29–36.

[5]Brentner, K., "Prediction of Helicopter Rotor Discrete Frequency Noise," NASA TR TM-87721, Oct. 1986.

[6]Ffowcs Williams, J. E., "The Acoustic Analogy—Thirty Years On," *Journal of Applied Mathematics*, Vol. 32, 1984.

[7]Kirchhoff, G. R., "Towards a Theory of Light Rays," *Annals of Physical Chemisty*, Vol. 18, 1883, pp. 663–695.

[8]Pierce, A. D., *Acoustics: An Introduction to Its Physical Principals and Applications*, Acoustical Society of America, New York, 1989, Chap. 5.

[9]Farassat, F., and Myers, M., "Extension of Kirchhoff's Formula to Radiation from Moving Surfaces," *Journal of Sound and Vibration*, Vol. 123, 1988, pp. 451–460.

[10]Betz, A., "Behavior of Vortex Systems," NACA TR TM-713, June 1933.

[11]Mason, W. H., and Marchman, J. F., III, "Far-Field Structure of Aircraft Wake Turbulence," *Journal of Aircraft*, Vol. 12, No. 2, 1973, pp. 86–92.

[12]Hoffmann, E., and Joubert, P. N., "Turbulent Line Vortices," *Journal of Fluid Mechanics*, Vol. 16, 1963, pp. 395–411.

[13]Rule, J., and Bliss, D., "Prediction of Turbulent Trailing Vortex Structure for Rotorcraft Blade-Vortex Interaction," *Proceedings of the 51st Annual Forum of the American Helicopter Society* (Fort Worth, TX), American Helicopter Society, Alexandria, VA, 1995.

[14]Milne-Thomson, L. M., *Theoretical Hydrodynamics*, Macmillan, New York, 1968, Chap. 13.

[15]Garrick, I. E., *Nonsteady Wing Characteristics, Aerodynamics of Aircraft Components at High Speed*, Vol. 7, Princeton Univ. Press, Princeton, NJ, 1957, Sec. F.

[16]Ashley, H., and Landahl, M., *Aerodynamics of Wings and Bodies*, Dover, Mineola, NY, 1965, Chap. 1.

[17]Morse, P. M., and Feshbach, H., *Methods of Theoretical Physics*, McGraw–Hill, New York, 1953.

[18]Kellogg, O. D., *Foundations of Potential Theory*, Dover, Mineola, NY, 1953, Chap. 2.

[19]Garrick, I. E., "On Moving Sources in Nonsteady Aerodynamics and in Kirchhoff's Formula," *Proceedings of the First U.S. National Congress of Applied Mechanics* (Chicago, IL), American Society of Mechanical Engineers, 1951.

[20]Morino, L., "A General Theory of Unsteady Compressible Potential Aerodynamics," NASA TR CR 2464, Langley Research Center, Dec. 1974.

[21]Larmor, J., "On the Mathematical Expression of the Principle of Huygens," *Proceedings of the London Mathematical Society*, Vol. 2, No. 1, 1903, pp. 1–13.

[22]Bliss, D. B., and Epstein, R. J., "A Novel Approach to Aerodynamic Analysis Using Analytical/Numerical Matching," *Proceedings of the 13th AIAA Applied Aerodynamics Conference* (San Diego, CA), AIAA, Washington, DC, 1995.

[23]Epstein, R. J., and Bliss, D. B., "An Acoustic Boundary Element Method Using Analytical/Numerical Matching," *Journal of the Acoustical Society of America*, Vol. 101, No. 1, 1997, pp. 92–106.

[24]Bliss, D. B., and Epstein, R. J., "Novel Approach to Aerodynamic Analysis Using Analytical/Numerical Matching," *AIAA Journal*, Vol. 34, No. 11, 1996, pp. 2225–2232.

[25]Epstein, R. J., and Bliss, D. B., "An Aeroacoustic Boundary Element Method Using Analytical/Numerical Matching," *Proceedings of the 1st Joint CEAS/AIAA Aeroacoustics Conference* (Munich, Germany), 1995.

[26]Epstein, R. J., and Bliss, D. B., "Aeroacoustic Boundary Element Method Using Analytical/Numerical Matching," *AIAA Journal*, Vol. 35, No. 2, 1997, pp. 244–254.

S. Fleeter
Associate Editor

Box-Length Requirements for Simulation of Sound from Large Structures in Jets

E. J. Avital* and N. D. Sandham†
Queen Mary and Westfield College, London E1 4NS, England, United Kingdom

Introduction

LARGE-SCALE structures in the mixing region of jets are known to be dominant sound producers in forced jets.[1,2] These structures can be modeled as instability waves, initially growing, then saturating and decaying. Thus one can refer to them as wave packets. This kind of model was used to determine the magnitude of the radiated sound.[3] It was also used to explain the appearance of superdirectivity in the sound field of low subsonic jets,[4] as had been found experimentally.[2] When it comes to simulating the sound generated by a large-scale structure, one problem is that one is confined to the use of a finite-size computational box (see Fig. 1). In this study it is assumed that the large-scale structure of the shear layer has been successfully simulated inside the finite box. Then the question is whether the box is long enough in the longitudinal direction to include a sufficient portion of the acoustic source to get an accurate description of the sound field. The answer depends partly on the quality of the boundary conditions applied on the box sides. However, we assume that the boundary conditions are so accurate that no distortion is created in the sound field due to them. Thus this study gives the minimum required computational length scale that is determined only by the physical mechanism of the sound generation.

Only two attempts to use a wave-packet model to determine the required computational box are reported in the literature. The first,

Received Oct. 26, 1996; revision received Jan. 15, 1997; accepted for publication Jan. 17, 1997.

*Ph.D. Student, Department of Engineering, Mile End Road. Student Member AIAA.

†Lecturer, Department of Engineering, Mile End Road. Member AIAA.